

UNIT #2 - MEASUREMENTS AND CALCULATIONS

SECTION 3: CALCULATIONS

(continued)

Multiplying and Dividing

After you add or subtract, you look at precision in order to round the answer.

A different rule applies when multiplying or dividing. Here you need to count the number of SIGNIFICANT FIGURES. Round off the answer to match the input number that has the fewest digits.

Example #1: A cube shaped box has these dimensions. Find its volume.

Length = 22.46 cm, Width = 13.2 cm, Height = 15.429 cm.

$$\text{Volume} = 22.46 \text{ cm} \times 13.2 \text{ cm} \times 15.429 \text{ cm} = 4,574.266488 \text{ cm}^3$$

(unrounded answer)
(what is shown on the calculator)

When you are given measurements you cannot assume that they are exact. The stated length is 22.46 cm. So we have to assume that this measurement is only good to the second decimal place. In other words '22.46' only has four significant figures.

Likewise 13.2 cm only has three significant figures, while 15.429 cm has five significant figures.

Therefore 13.2 is the 'worst' of the three input numbers, because it only has three sig. figs. Round the answer accordingly:

$$4,574.266488 \text{ cm}^3 \quad \rightarrow \quad 4,570 \text{ cm}^3 \quad (3 \text{ sig. figs})$$

(unrounded) (rounded)

****Important point - always remember to show the units of your answer.**

Example #2: Divide these two numbers:

$$444,444.4 \div 22,222.2200 = 20.000 \ 00$$

(7 sig.figs.) (9 sig.figs.) (Our 'worst' number has 7 digits, so we keep 7 digits in the answer.)

Example #3 - The same rules apply to scientific notation.

$$\begin{array}{ccc} (4.27 \times 10^6) \div (3.008 \times 10^2) = 1.419\,547\,872 \times 10^4 \\ \text{3 sig.figs.} \qquad \qquad \text{4 sig.figs.} \qquad \qquad \text{Unrounded Answer} \end{array}$$

One of the input numbers only has three significant figures. Therefore the answer must be rounded to three digits - 1.42×10^4 .

Example #4 - Calculating density.

To chemists, density is a very important property.

DENSITY = MASS \div VOLUME

Given: The mass of the rock is 139.2 kg.

The rock's volume is 24.119 Litres.

Find: The rock's density.

Density = Mass \div Volume

↓

Plug in numbers

↓

$$\begin{array}{ccc} \text{Density} = (139.2 \text{ kg}) \div (24.119 \text{ L}) = 5.771\,383\,557 \text{ kg/L} \\ \text{4 sig.figs.} \qquad \text{5 sig.figs.} \qquad \qquad \text{rough answer} \end{array}$$

↓

Round to four digits

↓

ANSWER = 5.771 kg/L

****Note the units. For density you always end up with units of mass per units of volume. ****

Example #5 - Lions at the zoo.

Given: The zoo contains 8 cages.

Each cage contains 6 lions.

Find: The total number of lions in the zoo.

Number of lions = (8 cages) x (6 lions per cage) = 48 lions

Is 48 an acceptable answer ?
Should it be rounded ?

Only one digit is shown in each of the input numbers ('8' and '6').
Would it make sense to round 48 to ONE digit ?
Is **50** the correct answer ?

In this case we are NOT dealing with measurements. Instead we have exact quantities. When we say that there are 6 lions per cage, there are exactly 6.0000 lions. You cannot have 6.3 or 5.72 lions. Lions only come in whole numbers. You can't have fractions.

Therefore 6 lions has more than one significant figure. Since it is exactly 6.0000, it has an infinite number of sig. figs.

The same is true of the cages. 8 cages are exactly 8.

In this case it does not make sense to round the answer to 50. 48 is the correct number of lions.

PERCENT UNCERTAINTIES

Percentages can tell us about the quality of our numbers. A large percent uncertainty indicates that the uncertainty (doubt) is big compared to the measurement. The quality of the measurement is poor.

To calculate a percent uncertainty, divide the uncertainty by the main number. Next multiply by 100 in order to convert it into a percentage.

Example: Measurement = 4.8 ± 0.9 cm

Uncertainty always has one digit

Percent Uncertainty = $(\pm 0.9 \text{ cm} \div 4.8 \text{ cm}) \times 100$

Percent Uncertainty = $\pm 20\%$
One digit

Clearly, an uncertainty as large as 20% raises serious doubt about the reliability of this measurement.

Converting From Scientific Notation To Long Form

Given: $3.24 \times 10^{-2} \pm 0.07 \times 10^{-2}$

To do: 1. Convert this number to 'long form'
2. Calculate the Percent Uncertainty

Converting To Long Form

Here the power is 10^{-2} . This means that the number starts in the second decimal place.

$$\begin{array}{ccc} 3.24 \times 10^{-2} & \pm & 0.07 \times 10^{-2} \\ \downarrow & & \downarrow \\ \text{Put in 2}^{\text{nd}} \text{ decimal place} & & \text{Put in 2}^{\text{nd}} \text{ decimal place} \\ \downarrow & & \downarrow \\ 0.0324 \pm 0.0007 & & \text{Long Form Notation} \end{array}$$

Calculating Percent Uncertainty

$$\text{Percent Uncertainty} = \left(\frac{\pm 0.0007}{0.0324} \right) \times 100 = \pm 2.1604938\%$$

However we only have one significant figure in the uncertainty (0.0007). Therefore round the answer to one digit.

ANSWER - Percent Uncertainty = $\pm 2\%$